Exercise 3

For each of the following integro-differential equations, classify as Fredholm, Volterra, or Volterra-Fredholm integro-equation

$$u''(x) + u(x) = x + \int_0^x tu(t) \, dt + \int_0^1 u(t) \, dt, \ u(0) = 0, \ u'(0) = 1$$

Solution

There is both a Volterra integral (one with a variable limit of integration) and a Fredholm integral (one with constant limits of integration) in the equation, so it is a Volterra-Fredholm integro-differential equation.